Correction of Doppler Radar Data for Aircraft Motion Using Surface Measurements and Recursive Least-Squares Estimation

S. L. Durden and Z. S. Haddad

Jet Propulsion Laboratory

California Institute of Technology

Pasadena, CA 91109

IEEE Geosci. Remote Sensing

Abstract

Observations of Doppler velocity of **hydrometeors** from airborne **Doppler weather radars** normally contains a component due to the **aircraft motion**. Accurate hydrometeor velocity measurements thus require correction by subtracting this velocity from the observed velocity. A method for estimating the aircraft motion contribution using the observed Doppler for the land or ocean surface is described. The method is based upon a **recursive least squares** fit of a model for the surface Doppler to data acquired over several scans of the radar antenna. Tests of the method on both simulated and real data show good performance when compared with simple subtraction of the observed surface Doppler from the observed hydrometeor Doppler velocities.

meteorological radar

1 Introduction

Airborne Doppler weather radars can provide unique data in situations that are not accessible to ground-based radars, such as hurricanes and oceanic mesoscale systems [1],[2],[3]. One of the challenges in measuring the velocity of hydrometeors using airborne Doppler radar is correction of the data for aircraft motion. One approach is use of accurate in situ measurements of aircraft orientation and motion. This can be used to estimate the antenna pointing and, hence, the contribution of the aircraft motion to the observed Doppler velocities. A second approach is estimation of the antenna pointing directly from the radar data. This approach has the advantage of not requiring auxiliary data with possible time offsets from the radar data. It also does not require accurate knowledge of the antenna mounting relative to the aircraft. This approach is often used in determining the processing parameters for airborne and spaceborne synthetic aperture radar (SAR) data [4]. It is the purpose of this paper to apply pointing estimation from surface Doppler data to processing data from airborne Doppler weather radars, specifially the NASA/JPL Airborne Rain MApping Radar (ARMAR), which is a 14 GHz multiple-polarization Doppler radar operating on the NASA DC-8 aircraft [3]. The technique developed here is novel in that it makes use of recursive least squares to incorporate data from several radar scans to estimate pointing parameters. We present in Section 2 the theory of the method, followed in Section 3 by demonstration of the method on simulated data and data acquired by ARMAR.

2 Description of the Method

The primary objective of the radar's Doppler capability is measurement of the component of the hydrometeor velocity vector in the radar look direction $\hat{k}$, where $\hat{}$ denotes a unit vector. If the platform moves with vector velocity $\overline{v_p}$, the radar measures $\hat{k} \cdot \overline{v_h} + \hat{k} \cdot \overline{v_p}$. Correction requires that $\hat{k} \cdot \overline{v_p}$ be estimated and subtracted from the measured velocities. Assuming that the land or ocean surface is stationary, a measurement of the surface within the same beam as the hydrometeors provides $\hat{k} \cdot \overline{v_p}$. The simplest approach to using the surface measurement is to simply subtract the measured surface Doppler from all range bins. The disadvantage of this approach is that the surface Doppler estimate in a single beam has the same accuracy as that in all the other range bins. If the errors at each bin are independent, the variance of the Doppler estimate is thus doubled by the correction procedure. This is generally undesirable, and a better approach is to use measurements at multiple scan angles to estimate the aircraft orientation and motion.

To formulate the correction procedure we derive a model of the measured surface Doppler given the aircraft orientation and the antenna pointing angles relative to the aircraft. In the antenna frame of reference the antenna pointing direction is specified by the unit vector $\hat{k}$, which can be written in terms of azimuth and elevation angles as:

$$\hat{k'} = \sin\theta \hat{x'} + \sin\phi\cos\theta \hat{y'} + \cos\phi\cos\theta \hat{z'} \tag{1}$$

By performing coordinate rotations, this vector can be transformed into global coordinates (relative to the earth's surface), in which $\hat{x}$ points along the true aircraft heading and $\hat{z}$ points toward nadir:

$$\hat{k} = \begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \cos p & 0 & \sin p \\ 0 & 1 & 0 \\ -\sin p & 0 & \cos p \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos r & -\sin r \\ 0 & \sin r & \cos r \end{bmatrix} \hat{k'} \tag{2}$$

where r, p, and y are the aircraft roll, pitch, and yaw, respectively. The aircraft velocity in the global coordinates is $\overline{v_p} = v_x\hat{x} - v_z\hat{z}$, where v_x is the aircraft ground speed and v_z is the aircraft vertical speed, with v_z positive for upward motion. The measured Doppler is defined to be positive for targets moving away from the radar, so the measured Doppler v_d is defined as $-\hat{k} \cdot \overline{v_p}$, or

$$v_d = -v_x \cos y \cos p \sin\theta - v_z \sin p \sin\theta - v_x \cos y \sin p \sin r \cos\theta \sin\phi + v_x \sin y \cos r \cos\theta \sin\phi$$

$$+v_z \cos p \sin r \cos\theta \sin\phi - v_x \cos y \sin p \cos r \cos\theta \cos\phi - v_x \sin y \sin r \cos\theta \cos\phi$$

$$+v_z \cos p \cos r \cos\theta \cos\phi \tag{3}$$

This equation provides an exact model for the measured doppler; it can be fit to measurements using nonlinear least squares, providing an estimate of the aircraft orientation and velocity.

To avoid using nonlinear least squares (3) can be written as a linear combination of trigonometric functions of ϕ and θ:

$$v_d = w_0 \sin\theta + w_1 \cos\theta \sin\phi + w_2 \cos\theta \cos\phi \tag{4}$$

In vector form $v_d = \overline{w} \cdot \overline{u}$, where $\overline{w}$ is the coefficient vector and $\overline{u}$ is the vector consisting of trigonometric functions of ϕ and θ. Given the Doppler measurements v_d for a set of ϕ and θ, an estimate of $\overline{w}$ can be found by linear least squares. These can then be used in (4) to estimate the velocity correction for each beam, which is then subtracted from all range bins within the beam. This approach is used operationally in correcting ARMAR Doppler data for platform motion and generally works well. However, it make use of observations over one scan only. Assuming that the platform parameters change slowly relative to the scan time, a more accurate estimate should be achievable using observations from several scans. This can be accomplished using recursive least squares (RLS), whereby the existing least squares solution is updated when new information is available.

The RLS problem has been studies in detail in the area of adaptive filtering, and both the theory and applications of adaptive filtering are described in detail by Haykin [5]. RLS finds the $\overline{w}$ at time n which minimizes the cost function $E(n) = \Sigma\lambda^{n-i}|v_d(i) - \overline{w(n)} \cdot \overline{u(i)}|^2$, where the sum is over all measurements up to time n. The factor $\lambda < 1$ is a forgetting factor so that the effect of data in the distant past is much less than that for recent data. The general form for the RLS solution is:

$$\overline{w(n)} = \overline{w(n-1)} + \overline{K(n)}(v_d(n) - \overline{w(n-1)} \cdot \overline{u(n)}) \tag{5}$$

where $K(n)$ is a gain vector at time step n. The equations for updating K are (Haykin 1991):

$$\overline{K(n)} = \alpha P(n-1)\overline{u(n)}/(1 + \alpha\overline{u(n)} \cdot P(n-1)\overline{u(n)} \tag{6}$$

$$P(n) = \alpha P(n-1) - \alpha\overline{K(n)} \cdot \overline{u(n)}P(n-1) \tag{7}$$

The matrix P is initialized to the identity matrix in our computations.

It should also be noted that adaptive filters can be designed which do not find the exact least squares solution at each time step. Instead, computationally simpler gradient-based search algorithms can also be used. These algorithms result in an update equation similar to (5). In this case $K(n)$ is replaced by $\lambda\overline{u}$; this is referred to as the least mean square (LMS) algorithm. It also should be noted that the RLS solution can also be viewed as a Kalman filter. As shown by Haykin use of the following state space model for the data

$$\overline{w(n)} = \overline{w(n-1)} + \overline{m(n)} \tag{8}$$

$$v(n) = \overline{w(n)} \cdot \overline{u(n)} + q(n) \tag{9}$$

leads to Kalman filter equations that are equivalent to (5)- (7). In this model $\overline{m(n)}$ and $q(n)$ are the plant and observation noise, respectively.

There are at least two assumptions implicit in using the Doppler correction method just described. First, the technique assumes that the aircraft orientation and velocity, or equivalently the vector $\overline{w}$ remains constant over some time longer than a single antenna scan. This is probably a reasonable assumption for ARMAR and the NASA DC-8; ARMAR scans are completed in 1.8 s. Second, it assumes that the mean surface velocity is the same as the required correction velocity. This requirement may not be met if the surface backscatter is not uniform within the radar resolution cell or if the surface is moving. Uniformity of backscatter is generally not likely; there are usually some targets within an 800 m resolution cell (in ARMAR's case), especially over land. In general, these are randomly located and do not cause a bias over multiple beams. However, a very smooth surface, such as the ocean at low wind speeds may be consistenly brighter in that part of the resolution cell closer to nadir, causing the surface Doppler to be smaller than the required correction. For ARMAR in typical ocean conditions, this bias is expected to be less than 1 m/s. The bias would be smaller for radars with larger antennas and/or on slower platforms.

3 Results

In this section we present the results of testing both the standard ARMAR algorithm, which performs a least squares solution for each scan, and the RLS estimation algorithm, which is equivalent to a Kalman filter. The first algorithm is noted as LS, while the second is RLS. Fig. 1 shows the result of applying algorithms to simulated data, generated by using (4) to compute sets of observed velocities corresponding to $\pm 20^o$ scans in the cross-track direction, analogous to ARMAR's observational geometry. The elevation angle θ is constant as is normally the case in ARMAR data. The simulated data correspond to 1000 simulated scans acquired with ground speed of 225 m/s, with 1 m/s upward motion, 1^o roll, 1^o pitch, and 5^o yaw. A Gaussian measurement noise of 0.4 m/s was added to each observation; this is typical for ARMAR using a pulse-pair Doppler estimator. Fig. 1 shows surface Doppler velocity estimates for $\phi = 0^o$ (closest antenna position to nadir); estimation accuracies for other ϕ are essentially identical to those for $\phi = 0^o$. Velocity estimates shown are the observed noisy Doppler at nadir and LS and RLS velocity estimates. The true $\phi = 0^o$ velocity is -2.57 m/s. The worst estimate of the true velocity is the observed nadir velocity. The root mean square (RMS) difference between the observed surface velocity and the true is 0.4 m/s, i.e. the measurement noise. Using the full scan to perform the estimate (LS algorithm) results in a RMS difference of 0.13 m/s, much better than using a single observation. The RLS algorithm with $\lambda = 0.98$ reaches steady state in about 5 scans, with the RMS difference reduced to 0.06 m/s. The RLS case with $\lambda = 1.00$ does not reach steady state; its estimate continues to improve since it makes use of all past data. These two cases for RLS illustrate the tradeoff in choosing λ; if λ is too close to unity, the final accuracy is higher but the adaptation time may be very large. λ must be chosen small enough that the filter adapts to changes over time but not so small that the estimate is based on too little data.

Fig. 2a displays the result of applying the algorithms to 250 contiguous ARMAR scans over the ocean during a straight flight track. Shown are both the observed velocity at nadir and the velocity estimated using the RLS algorithm with $\lambda = 0.98$. The RLS estimate shows much less variability than the observed velocity. Fig. 2b shows the nadir velocity estimated from the DC-8 navigational data over the same time, using (4). Comparison of Fig 2a with Fig. 2b, shows that the radar and aircraft velocity measurements show the same general behavior; however the aircraft velocity has much more variability, possibly due to measurement errors in the navigational parameters.

4 Conclusions

A method for correcting Doppler observations from airborne weather radars has been described. The method does not use aircraft navigational data and is therefore not subject to errors in navigational parameter measurement, time offsets between aircraft and radar measurements, and uncertainties in the relation of the antenna pointing to the aircraft orientation. The method described here uses the radar observations of the Doppler associated with the surface to correct the observations of hydrometeors. The method is formulated in terms as a linear estimation problem which is solved by recursive least squares. The method was tested on both simulated data and data from ARMAR and was found to produce good results, as compared with simple subtraction of the observed surface Doppler from the observed hydrometeor Doppler velocities. It appears that the method produces reasonable corrections for aircraft motion, given the assumptions stated in Section 2.

Acknowledgment

The research described here was performed by the Jet Propulsion Laboratory, California Institute of Technology, under contract with the National Aeronautics and Space Administration. Support from the NASA Atmospheric Dynamics and Remote Sensing program is gratefully acknowledged.

References

1. P. H. Hildebrand and C. K. Mueller, "Evaluation of meteorological airborne Doppler radar. Part I: Dual-Doppler analyses of air motions,", J. Atmos. Oceanic Technol., vol. 2, pp. 362-380.

2. P. H. Hildebrand, C. A. Walther, C. L. Frush, J. Testud, and F. Baudin, "The ELDORA/ASTRAIA airborne Doppler weather radar - goals, design, and first field tests, " *Proc. IEEE*, vol. 82, no. 12, pp 1873-1890, 1994.

3. S. L. Durden, E. Im, F. K. Li, W. Ricketts, A. Tanner, and W. Wilson, "ARMAR: An airborne rain mapping radar, " *J. Atmos. Oceanic Tech.*, vol. 11, pp. 727-737, 1994.

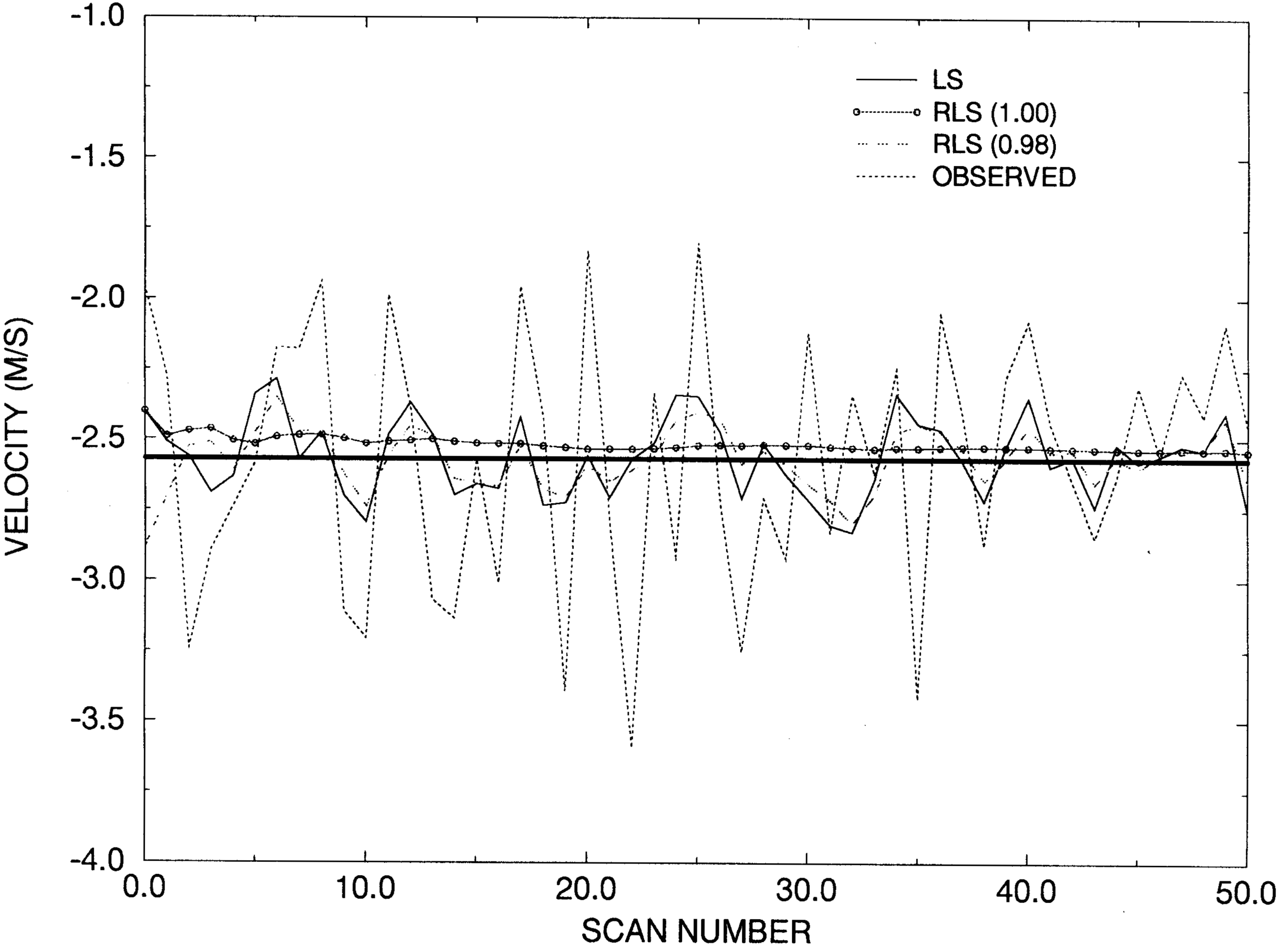
4. J. C. Curlander and R. N. McDonough, *Synthetic Aperture Radar.*

5. S. Haykin, *Adaptive Filter Theory.*

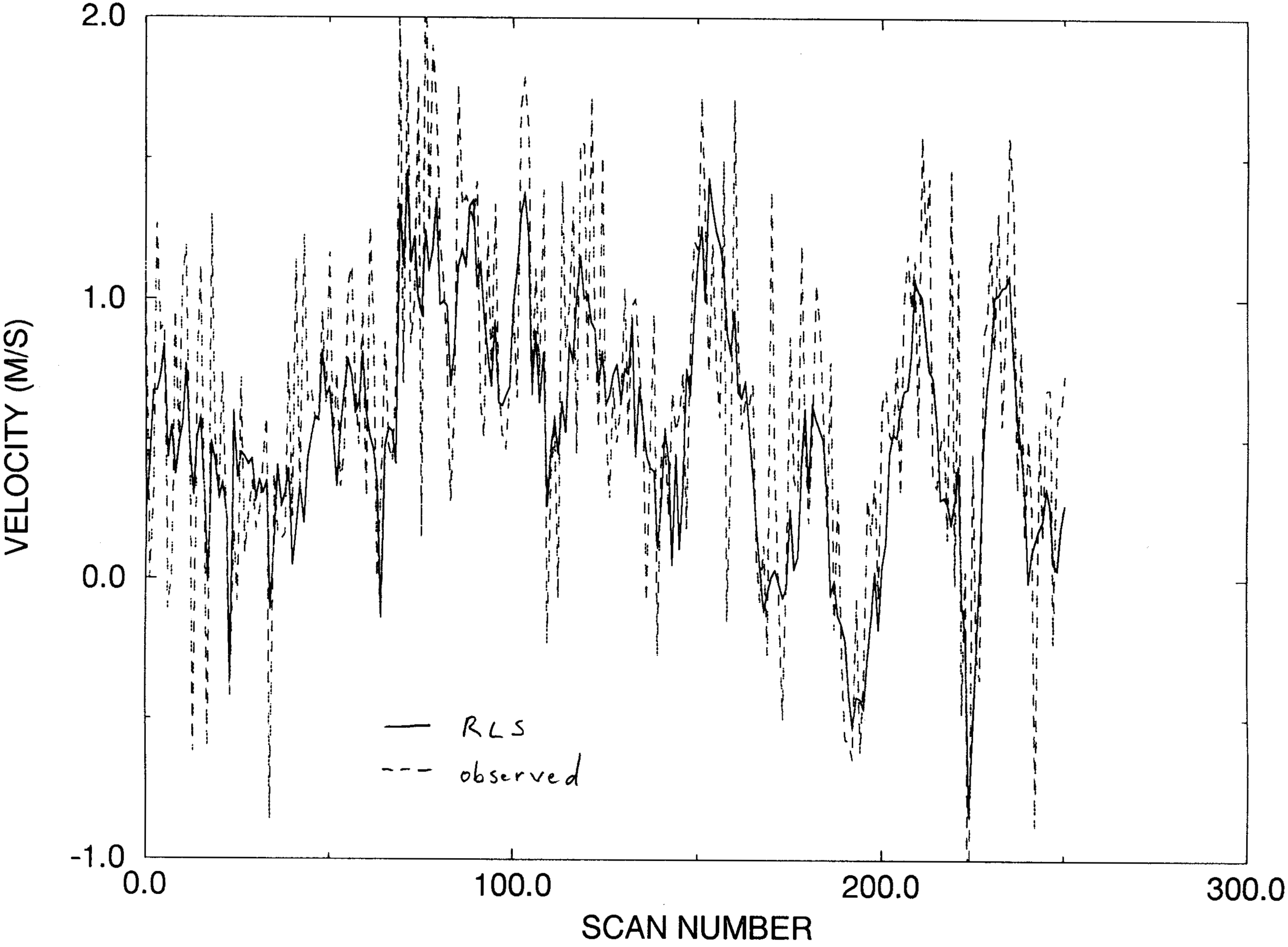
Figure Captions

Figure 1. Estimated velocity correction at $\phi = 0^o$ for simulated data.

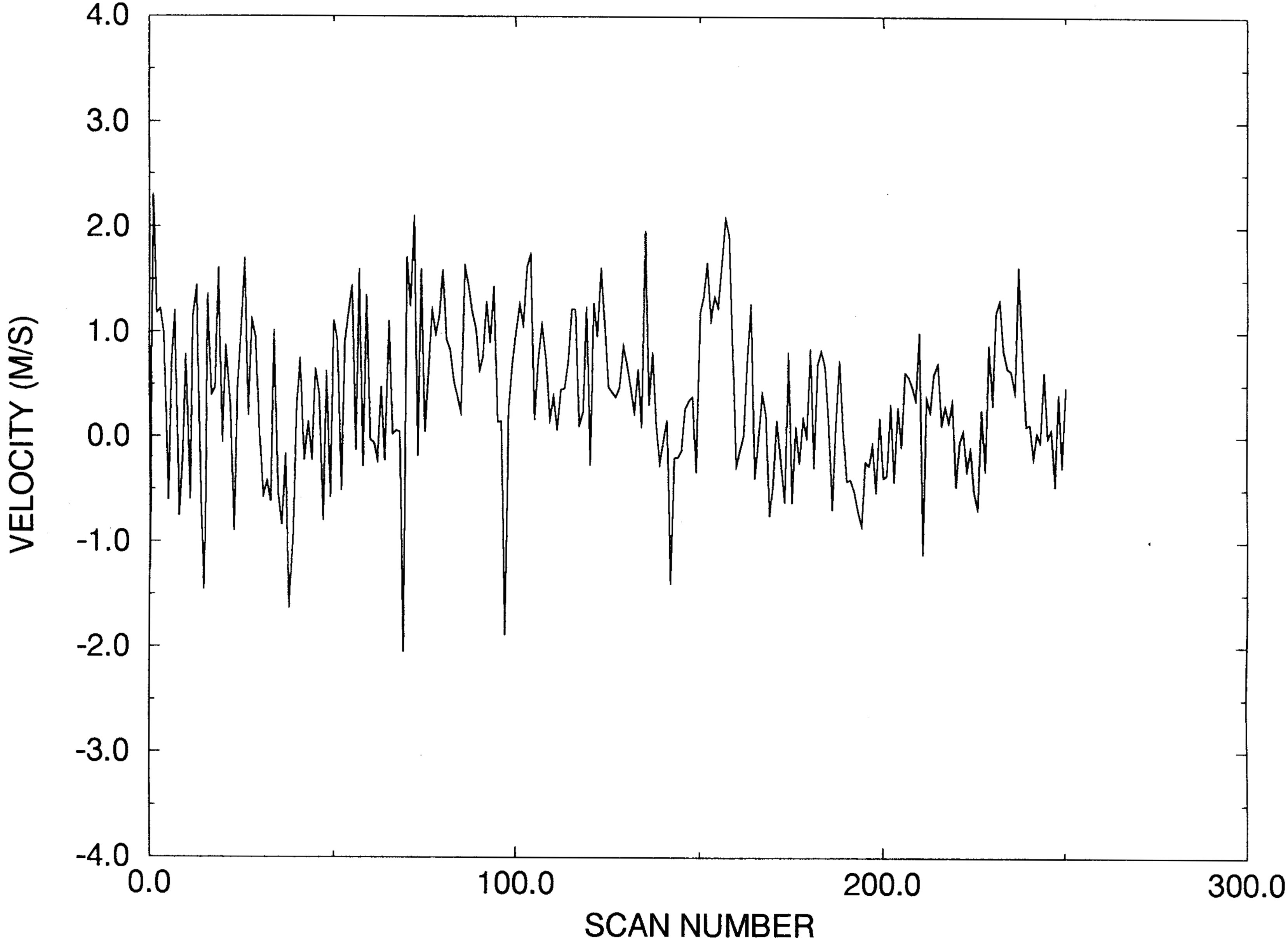
Figure 2. (a) RLS velocity and observed nadir velocity over 250 scans of ARMAR ocean data. (b) Velocity for same 250 scans using DC-8 navigational parameters.



LS
RLS (1.00)
RLS (0.98)
OBSERVED
VELOCITY (M/S)
SCAN NUMBER
-1.0
-1.5
-2.0
-2.5
-3.0
-3.5
-4.0
0.0
10.0
20.0
30.0
40.0
50.0



VELOCITY (M/S)
2.0
1.0
0.0
-1.0
0.0
100.0
200.0
300.0
SCAN NUMBER
RLS
observed


VELOCITY (M/S)
4.0
3.0
2.0
1.0
0.0
-1.0
-2.0
-3.0
-4.0
0.0
100.0
200.0
300.0
SCAN NUMBER